Random trees and their scaling limits

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In the last 30 years, random combinatorial structures and their limits have been a flourishing area of research at the interface between probability and combinatorics. In this course, I hope to show you some of the beautiful theory that arises when considering scaling limits of random trees.

Trees are fundamental objects in combinatorics and the enumeration of different classes of tree is a classical subject. We will take as our basic object the genealogical tree of a Galton-Watson branching process. (As well as having nice probabilistic properties, this class turns out to include various natural types of random combinatorial tree in disguise.) In the same way as Brownian motion is the universal scaling limit for centred random walks of finite step-size variance, it turns out that all critical Galton-Watson trees with finite offspring variance have a universal scaling limit, Aldous' Brownian continuum random tree (CRT). (In fact, this is not just an analogy!) The CRT is a so-called R-tree, a path metric space which is connected and has no cycles. In the first section of this course, I will discuss the convergence to the CRT (and what exactly we mean by convergence of a tree). Then I will move on to consider what happens in the infinite variance case, where different objects (the stable trees, introduced by Duquesne, Le Gall and Le Jan) arise in the limit. In the final part of the course (time-permitting) I will talk about some interesting characterisations of the limit objects.